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An Application of Noninteracting Control to a Continuous Flow Stirred-Tank Reactor

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A general method, given in a companion paper, by which an interacting linear multivariable system may be decoupled into independent subsystems, is applied to a continuous flow stirred-tank reactor. The form of the compensating controllers required for the physical system is obtained. It is then verified numerically that noninteraction is achieved, as postulated, by simulation of the linear model with compensating control on the analog computer.

The characteristic property of a multivariable system is that each of its inputs will generally affect more than one output simultaneously. Conventional control of such an interacting system can be both difficult and inefficient in that each control device must be compromised in its design. Although primary emphasis is placed on achieving adequate control of a corresponding output, care must be taken that the controller will not adversely affect the remaining system outputs. This difficulty has been eliminated in a companion paper (*1*) by describing theoretically the design of a compensating control device of generally simple form which may be applied to linear multivariable systems of any order. The technique makes it possible to break the system down into independent subsystems containing a single output as a function of a single manipulatable input and a single measurable input. Final output control of each subsystem may then be achieved in the absence of undesirable system interactions.

It is the purpose of this paper to apply the compensating technique described in reference *1* to a particular physical system, the continuous flow stirred-tank reactor. In addition to describing the form of the compensating controllers required for the given system, it will be verified numerically by means of analog computer simulation that noninteraction is achieved as postulated.

DESCRIPTION OF THE SYSTEM

The general theory of noninteracting control that has been developed (*1*) will be applied to a continuous flow stirred-tank reaction which contains a material undergoing a reaction of the type $X \xrightarrow{k}$ products, for which rate of conversion of $X(t)$ is given by $dX(t)/dt = kX(t) = A' e^{-E/RT(t)} X(t)$. The system, as described by Kermode and Stevens (*3*), is pictured in Figure 1.

The unsteady state heat and mass balance characterizing the system may be written as follows:

$$\frac{dX(t)}{dt} = \frac{Q(t)}{V} [X_i(t) - X(t)] - A' e^{-E/RT(t)} X(t) \qquad (1)$$

$$\frac{dT(t)}{dt} = \frac{Q(t)}{V} [T_i(t) - T(t)] - \frac{UAF(t)\,[T(t) - T_c]}{V_p C_p\,[F(t) + 1]} - \frac{A' e^{-E/RT(t)} \Delta H X(t)}{\rho\, C_p} \qquad (2)$$

$$F(t) = \frac{2\, Q_c(t)\, \rho_c\, C_c}{UA}$$

The system will be controlled about its unstable point at

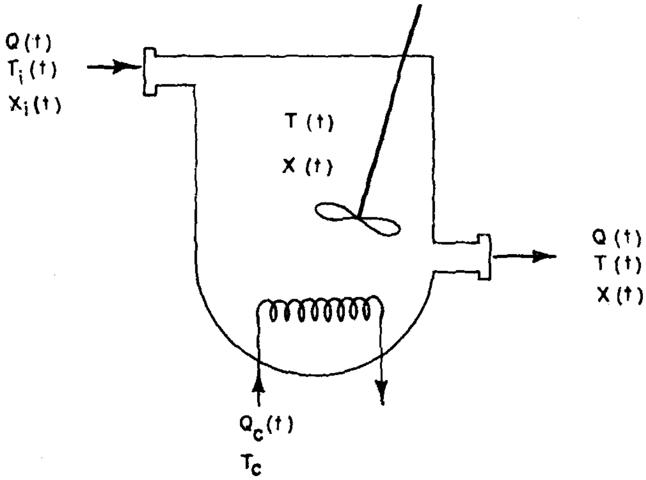

Fig. 1. Schematic diagram of system studied.

the values of system parameters and steady state inputs and outputs given in the Notation section. System inputs and outputs for all work to follow are defined as follows:

$$\left.\begin{array}{l} y_1(t) \equiv \overline{X}(t) = X(t) - X_o \\ y_2(t) \equiv \overline{T}(t) = T(t) - T_o \end{array}\right\} \text{system outputs}$$

$$\left.\begin{array}{l} x_1(t) \equiv \overline{Q}(t) = Q(t) - Q_o \\ x_2(t) \equiv \overline{Q}_c(t) = Q_c(t) - Q_{co} \end{array}\right\} \text{manipulatable inputs}$$

$$\left.\begin{array}{l} x_3(t) \equiv \overline{T}_i(t) = T_i(t) - T_{io} \\ x_4(t) \equiv \overline{X}_i(t) = X_i(t) = X_{io} \end{array}\right\} \text{measurable inputs}$$

With the above notation, Equations (1) and (2) are linearized and Laplace transformed with the $\overline{x}_i(s)$ and $\overline{y}_i(s)$ replaced by x_i and y_i for convenience to give

$$R_{11}y_1 = K_{11}x_1 + K_{14}x_4 + N_{12}y_2 \tag{3}$$

$$R_{22}y_2 = K_{21}x_1 + K_{22}x_2 + K_{23}x_3 + N_{21}y_1 \tag{4}$$

where

$$R_{11} = s - N_{11} \qquad K_{22} = \frac{2\,\rho_c C_c\,[T_c - T_o]}{V\rho C_p\,[F_o + 1]^2}$$

$$R_{22} = s - N_{22} \qquad K_{23} = \frac{Q_o}{V}$$

$$K_{11} = \frac{X_{io} - X_o}{V} \qquad N_{11} = -\frac{Q_o}{V} - A'\,e^{-E/RT_o}$$

$$K_{14} = \frac{Q_o}{V} \qquad N_{12} = -\frac{A'\,EX_o e^{-E/RT_o}}{RT_o^2}$$

$$K_{21} = \frac{T_{io} - T_o}{V} \qquad N_{21} = -\frac{A'\,\Delta H\,e^{-E/RT_o}}{\rho C_p}$$

$$N_{22} = -\frac{Q_o}{V} - \frac{UAF_o}{V\rho C_p\,[F_o + 1]} - \frac{A'\,E\Delta H X_o\,e^{-E/RT_o}}{\rho C_p R T_o^2}$$

TABLE 1

Output	Form	Time/ cycle	Damp- ing ratio	Peak amplitude
$y_1(t) = \overline{X}(t)$	Damped oscillatory	30 sec.	0.5	1% of X_o
$y_2(t) = \overline{T}(t)$	Damped oscillatory	30 sec.	0.5	1% of T_o

Output characteristics and limits are now set for step changes in $x_3(t)$ and $x_4(t)$ in Table 1. These output forms were chosen as an arbitrary standard for purposes of analysis, although in practice such factors as product quality and cost would be considered.

STABLE, NONINTERACTING CONTROL OF THE LINEAR SYSTEM

The objective of this section is to apply the noninteracting control theory developed in reference 1 to the linear model of the given physical system. The procedure will be presented diagramatically rather than in matrix form so as to help make clear the physical form of the compensating control system. Equations (3) and (4) may be rewritten as

$$y_1 = P_{11}x_1 + P_{12}x_2 + P_{13}x_3 + P_{14}x_4 \tag{5}$$

$$y_2 = P_{21}x_1 + P_{22}x_2 + P_{23}x_3 + P_{24}x_4 \tag{6}$$

where

$$P_{ij} = \frac{f_{ij}(s)}{R_{11}R_{22} - N_{12}N_{21}}$$

The system of Equations (5) and (6) is referred to as the standard or P form and is presented diagramatically in Figure 2.

The first step in the decoupling procedure is to form partially noninteracting subsystems such that $y_1 = y_1(x_1, x_3)$ and $y_2 = y_2(x_2, x_3)$, which means that it is necessary to eliminate the effect of x_2 and x_4 on y_1 and x_4 on y_2. Noninteraction of this type is achieved by transformation of the system mathematically to the V' canonical form, which requires that the inputs to be eliminated are transformed first to outputs and then fed back internally to the input of the system. This requires the presence of outputs y_1, y_2, and y_4. Since no y_4 exists, it is introduced as a "virtual" output, as shown in Figure 3.

The augmented system of Figure 3 may now be transformed to the V' canonical form, which is represented diagramatically in Figure 4. Here y_1 is made a direct

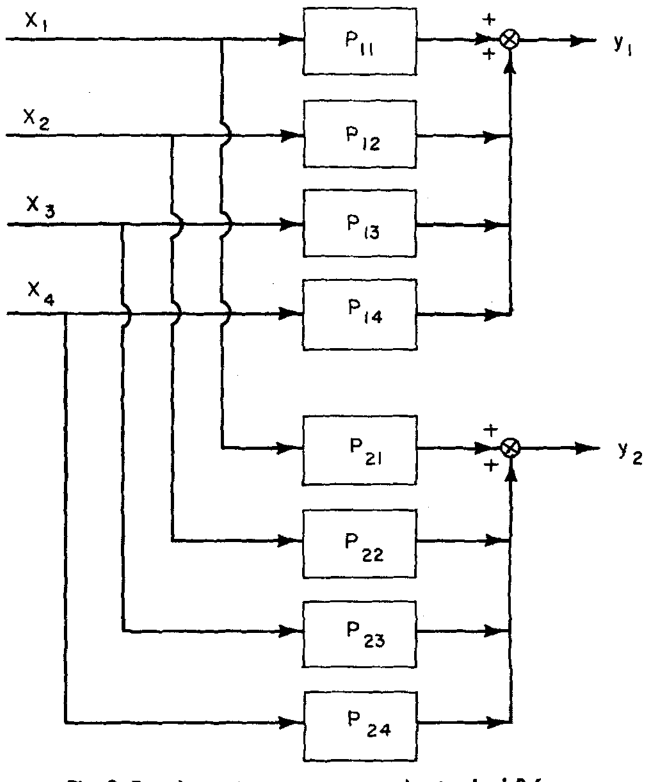

Fig. 2. Four-input, two-output system in standard P form.

function of x_1 and x_3 and a reverse function of x_2 and x_4. Similarly, y_2 is made a direct function of x_2 and x_3 and a reverse function of x_1 and x_4. Partial noninteraction is now easily accomplished by adding compensating controllers C_{12}, C_{14}, and C_{24} as shown in Figure 5.

If each C_{ij} is made equal to the negative of a corresponding V_{ij}, the following partially noninteracting system of Figure 6 will result. As shown in Figure 6, $y_1 = y_1(x_1, x_3)$ and $y_2 = y_2(x_2, x_3)$ as desired. The form of the F_{ij} and V_{ij} (and hence the C_{ij}) may be obtained from Equations (38) to (41) of reference 1 for $m = 2$. This yields

$$F_{11} = K_{22}K_{44}s - \frac{J_{33}}{[K_{14}K_{42}N_{21} + K_{22}K_{44}N_{11}]} \quad (7)$$

$$F_{13} = K_{22}K_{44}s - \frac{-J_{31}}{[K_{14}K_{42}N_{21} + K_{22}K_{44}N_{11}]} \quad (8)$$

$$F_{22} = \frac{J_{33}}{(K_{11}K_{44} - K_{14}K_{41})s + [K_{21}K_{44}N_{12} - (K_{11}K_{44} - K_{14}K_{41})N_{22}]} \quad (9)$$

$$F_{23} = \frac{-J_{32}}{(K_{11}K_{44} - K_{14}K_{41})s + [K_{21}K_{44}N_{12} - (K_{11}K_{44} - K_{14}K_{41})N_{22}]} \quad (10)$$

$$V_{12} = -C_{12} = -\frac{K_{14}K_{42}s + [K_{22}K_{44}N_{12} + K_{14}K_{42}N_{22}]}{J_{33}} \quad (11)$$

$$V_{21} = -C_{21} = \frac{K_{21}K_{44}s + [(K_{11}K_{44} - K_{14}K_{41})N_{21} - K_{21}K_{44}N_{11}]}{J_{33}} \quad (12)$$

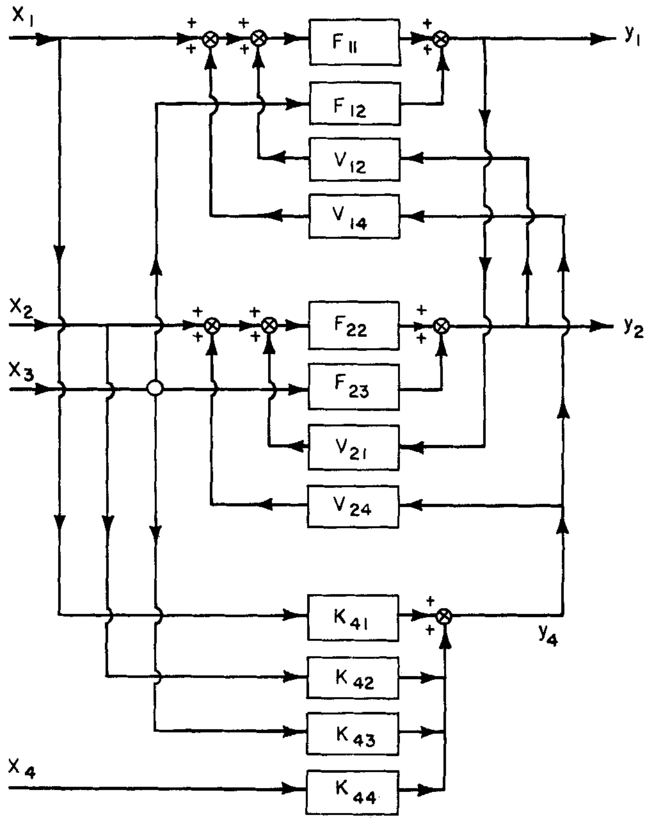

Fig. 4. Four-input, two-output system in V′ canonical form.

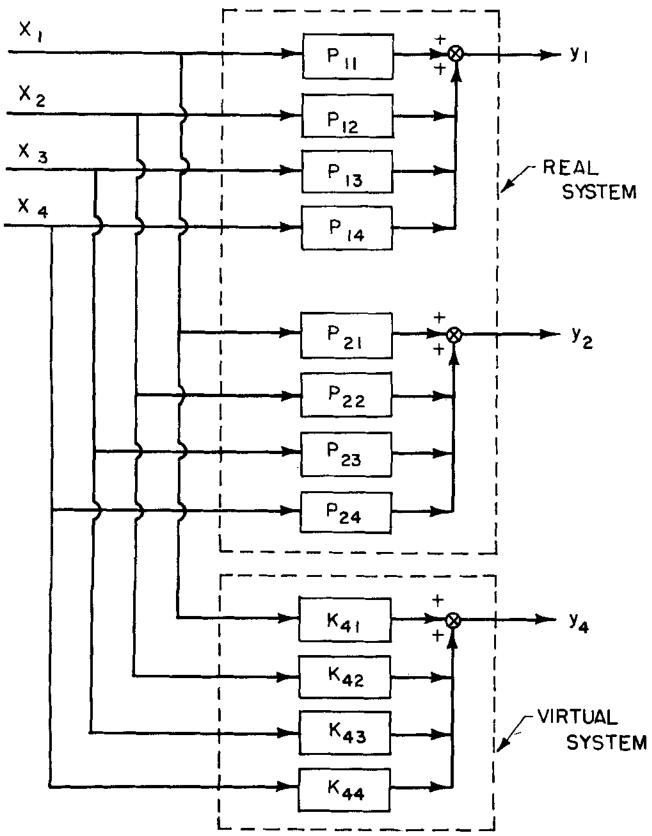

Fig. 3. Four-input, two-output system in standard P form, with virtual output added.

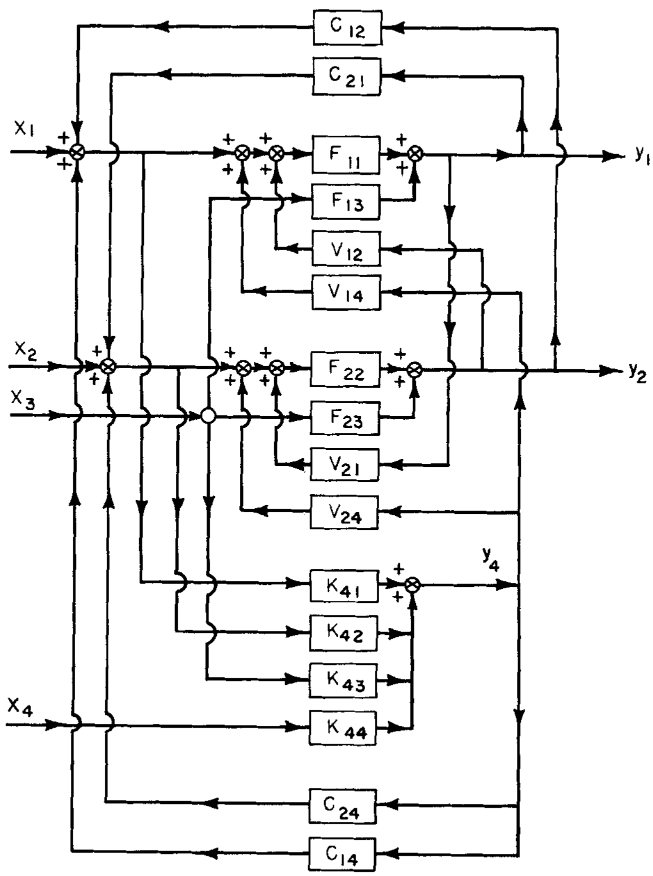

Fig. 5. Four-input, two-output system in V′ canonical form with compensating controllers added.

$$V_{14} = -C_{14} = \frac{K_{14}K_{22}}{J_{33}} \quad (13)$$

$$V_{24} = -C_{24} = -\frac{K_{14}K_{21}}{J_{33}} \quad (14)$$

Also, from the definition of J_{ij} given in reference 1, the values of J_{31}, J_{32}, and J_{33} are obtained:

$$J_{31} = K_{14}K_{22}K_{43} - K_{14}K_{23}K_{42} \quad (15)$$

$$J_{32} = K_{14}K_{23}K_{41} - K_{14}K_{21}K_{43} - K_{11}K_{23}K_{44} \quad (16)$$

$$J_{33} = K_{11}K_{22}K_{44} - K_{14}K_{22}K_{41} + K_{41}K_{21}K_{42} \quad (17)$$

The relation of the compensating controllers to the linear system in its physical P form may be seen by replacing the transfer functions F_{11}, F_{13}, F_{22}, F_{23}, V_{12}, V_{21}, V_{14}, and V_{24} in Figure 5 by the transfer functions P_{11}, P_{12}, P_{13}, P_{14}, P_{21}, P_{22}, P_{23}, and P_{24}. This final result is shown in Figure 7.

Figure 7 and Equations (11) to (14) show that the compensating device consists of feedforward amplifiers K_{41}, K_{42}, K_{43}, K_{44}, C_{14}, and C_{24} and feedback proportional plus derivative controllers C_{12} and C_{21}.

System stabilization and complete noninteraction is accomplished by adding a feedforward and a feedback controller to each of the subsystems of Figure 6. As an example, subsystem 1 under control is shown in Figure 8. For this system

$$y_1 = \frac{(F_{13} + F_{11}\ G_{ffi})}{1 + G_{fbi}\ F_{11}} x_3 \quad (18)$$

Similarly, for subsystem 2

$$y_2 = \frac{F_{23} + F_{22}\ G_{ff2}}{1 + F_{22}\ G_{fb2}} x_3 \quad (19)$$

Perfect control of y_1 and y_2 for any value of x_3 is accomplished by setting

$$G_{ff1} = -\frac{F_{13}}{F_{11}} \quad (20)$$

$$G_{ff2} = -\frac{F_{23}}{F_{22}} \quad (21)$$

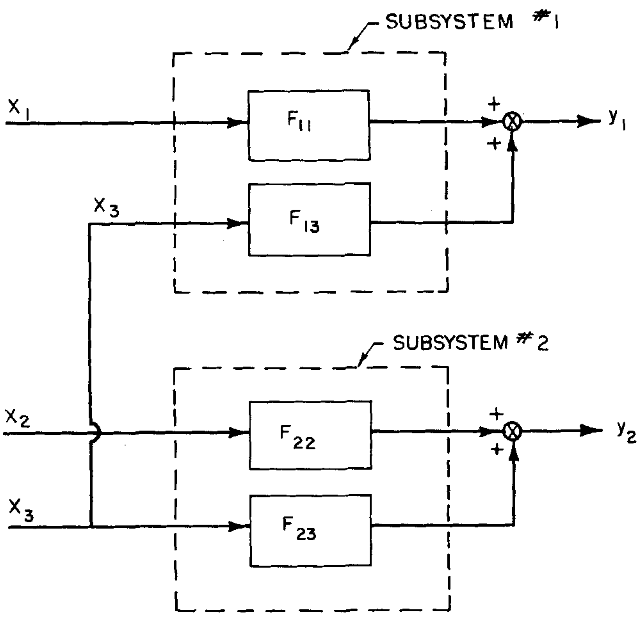

Fig. 6. Partially noninteracting subsystems resulting from addition of compensating controllers to four-input, two-output system in V' canonical form.

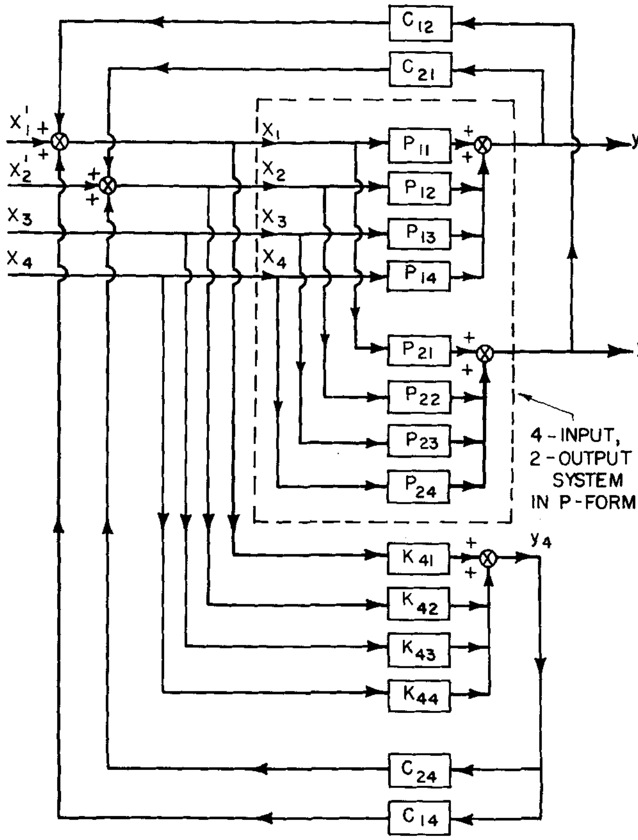

Fig. 7. Four-input, two-output system in standard *P* form with compensating controller added.

which, from Equations (7) to (10), becomes

$$G_{ff1} = \frac{J_{31}}{J_{33}} \quad (22)$$

$$G_{ff2} = \frac{J_{32}}{J_{33}} \quad (23)$$

Hence, G_{ff1} and G_{ff2} are simply feedforward amplifiers. Realization of perfect control is possible only if each subsystem is stable. This requires that the denominator terms of Equations (18) and (19), $1 + F_{11}\ G_{fbi}$ and $1 + F_{22}\ G_{fb2}$, have no right-half plane zeros. Stabilization of these denominator terms is a very simple matter due to the simple form of F_{11} and F_{22}. For generality, let G_{fb1} and G_{fb2} take on the most complex possible form, proportional-derivative-integral, whence

$$G_{fb1} = K_{C1}\left[1 + T_{D1}\ s + \frac{1}{T_1 s}\right] \quad (24)$$

$$G_{fb2} = K_{C2}\left[1 + T_{D2}\ s + \frac{1}{T_2 s}\right] \quad (25)$$

Insertion of Equations (24), (25), (7), (8), (9), and (10) into Equations (18) and (19) yields

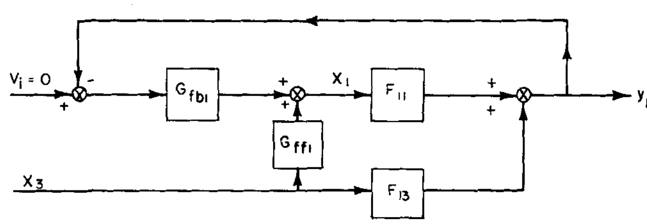

Fig. 8. Subsystem 1 with final control elements.

$$y_1 = \frac{\dfrac{T_1(J_{33}\ G_{ff1} - J_{31})s}{J_{33}\ K_{C1}}}{T_1^2\ s^2 + 2\,T_1\ \xi_1\ s + 1}\,x_3 \tag{26}$$

$$y_2 = \frac{\dfrac{T_2(J_{33}\ G_{ff2} - J_{32})s}{J_{33}\ K_{C2}}}{T_2^2\ s^2 + 2\ T_2\ \xi_2\ s + 1}\,x_3 \tag{27}$$

where

$$T_1^2 = \frac{T_1(K_{22}\,K_{44} + K_{C1}\,T_{D1}\,J_{33})}{K_{C1}\,J_{33}} \tag{28}$$

$$2\,T_1\,\xi_1 = \frac{T_1\,[K_{C1}\,J_{33} - (K_{14}\,K_{42}\,N_{21} + K_{22}\,K_{44}\,N_{11})]}{K_{C1}\,J_{33}} \tag{29}$$

$$T_2^2 = \frac{T_2\ (T_{D2}\,K_{C2}J_{33} + K_{11}\,K_{44} - K_{14}\,K_{41})}{K_{C2}\,J_{33}} \tag{30}$$

$$2\,T_2\,\xi_2 = \frac{T_2\ [K_{C2}\,J_{33} + K_{21}\,K_{44}\,N_{12} - (K_{11}\,K_{44} - K_{14}\,K_{41})\,N_{22}]}{K_{C2}\,J_{33}} \tag{31}$$

The quantities T_1 and ξ_i, representing characteristic time and damping ratio, respectively, may be set at any desired positive value in line with preset output characteristics hence guaranteeing subsystem stability.

In order to have nonzero output values for use in numerical comparisons to follow, the subsystem feedforward controllers will be omitted so that the subsystem equations to be considered become

$$Y_1 = \frac{\dfrac{-T_1\,J_{31}}{K_{C1}\,J_{33}}\,s}{T_1^2\,s^2 + 2\,T_1\,\xi_1\,s + 1}\,x_3 \tag{32}$$

$$Y_2 = \frac{\dfrac{-T_2\,J_{32}}{K_{C2}\,J_{33}}\,s}{T_2^2\,s^2 + 2\,T_2\,\xi_2\,s + 1}\,x_3 \tag{33}$$

Inversion of Equations (32) and (33) in the time domain for $x_3(t)$, a step function A_1, yields

$$y_1(t) = \frac{A_1\,T_1\,J_{31}}{T_1^2\,K_{C1}\,J_{33}\sqrt{1-\xi_1^2}}\,e^{-\frac{\xi_1 t}{T_1}}\sin\left(\frac{\sqrt{1-\xi_1^2}\ t}{T_1}\right) \tag{34}$$

$$y_2(t) = \frac{-A_1\,T_2\,J_{32}}{T_2\,K_{C2}\,J_{33}\sqrt{1-\xi_2^2}}\,e^{-\frac{\xi_2 t}{T_2}}\sin\left(\frac{\sqrt{1-\xi_2^2}\ t}{T_2}\right) \tag{35}$$

The output forms represented by Equations (34) and (35) are damped oscillatory as shown in Figure 9.

NUMERICAL VERIFICATION OF SYSTEM NONINTERACTION

For this system there are ten undetermined controller constants: K_{41}, K_{42}, K_{43}, K_{c1}, K_{c2}, T_{d1}, T_{d2}, T_1, and T_2. For the fixed output forms given in Table 1, T_1, T_2, ξ_1, and ξ_2 are fixed. It is also convenient to fix J_{33}, and so there are five restricting system equations to be solved in terms of five remaining undetermined constants, which may be set arbitrarily or adjusted in such a manner as to: (1) adjust output peak amplitudes to small values in the case where subsystem feed forward controllers are omitted or operate imperfectly, and (2) adjust manipulatable system inputs so that better correspondence is obtained between the linear model, which forms the basis

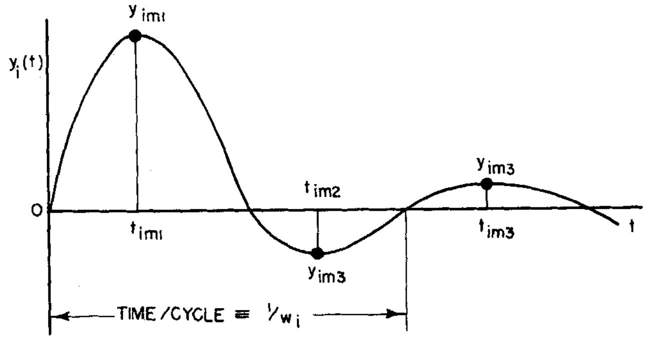

Fig. 9. Damped oscillatory output form.

for the design of the controllers, and the true nonlinear model. For the numerical work to be presented, the five undetermined constants T_{d1}, T_{d2}, K_{c1}, K_{c2}, and K_{43} will be set arbitrarily for simplicity. The values of these constants as well as J_{33} chosen are

$$\begin{aligned}
K_{c1} &= 1.0\text{ ft.}^6/(\text{lb.-moles})(\text{sec.})\\
K_{c2} &= 5.0\text{ cu.ft.}/(°\text{R.})(\text{sec.})\\
T_{d1} &= 1.0\text{ sec.}\\
T_{d2} &= 1.0\text{ sec.}\\
K_{43} &= 5.87\\
J_{33} &= 0.1
\end{aligned}$$

Final evaluation of $y_1(t)$ and $y_2(t)$, Equations (34) and (35), requires that $x_3(t)$ also be set or at least that its expected range be known, since it cannot be manipulated. For purposes of this analysis, $x_3(t)$ $(=\overline{T_i}(t))$ will be set at the step value $A_1 = 4°$R. From Equations (34) and (35), for the set output characteristics of Table 1, the obtained output parameters are shown in Table 2.

For the assigned values of K_{c1}, K_{c2}, T_{d1}, T_{d2}, and J_{33} and the values of T_1, T_2, ξ_1, and ξ_2 from Table 2, simultaneous solution of Equations (28) to (31) plus Equation (17) yields

$$\begin{aligned}
T_1 &= 0.0676\text{ sec.}\\
T_2 &= 4.30\text{ sec.}\\
K_{41} &= -321.7\\
K_{42} &= -641.1\\
K_{44} &= -46.86
\end{aligned}$$

For the above values of K_{c1}, K_{c2}, T_1, T_2, A_1, T_1, T_2, J_{33}, K_{41}, K_{42}, K_{43}, K_{44}, ξ_1, ξ_2, t_{1m1}, and t_{2m1}, Equations (15), (16), (34), and (35) may be combined to give

$$\begin{aligned}
y_{1m1} &= -0.000091\text{ lb.-moles/cu.ft.}\\
y_{2m1} &= -0.0036°\text{R.}
\end{aligned}$$

In summary, if noninteraction is accomplished as desired, the data in Table 3 may be compiled.

It may be verified that the system outputs have the above form independent of $x_4(t)$ (hence noninteracting) by means of analog computer simulation. This simulation was carried out on the Pace TR-10 analog computer by

TABLE 2. OUTPUT PARAMETERS CHARACTERIZING THE OUTPUT FORMS OF TABLE 1 FOR STEP INPUT $x_3(t)$

Parameter ($i = 1$ or 2)	Numerical value
$1/\omega_i$	30 sec.
ξ_i	0.5
t_{im1}	5 sec.
T_i	3.953
$\left\lvert\dfrac{y_{im1}}{y_{im2}}\right\rvert$	6.0

TABLE 3. OUTPUT CHARACTERISTICS OF IDEALLY DECOUPLED LINEAR MODEL REPRESENTING A CONTINUOUS FLOW STIRRED-TANK REACTOR FOR A GIVEN SET OF COMPENSATING AND STABILIZING CONTROLLER CONSTANTS

Controller constants

$K_{41} = -321.7$	$T_2 = 4.30$ sec.
$K_{42} = -641.1$	$T_{d1} = 1.0$ sec.
$K_{43} = 5.87$	$T_{d2} = 1.0$ sec.
$K_{44} = -46.86$	$K_{c1} = 1.0$ ft.^{6}/(lb.-moles)(sec.)
$T_1 = 0.0676$ sec.	$K_{c2} = 5.0$ cu. ft./(°R.)(sec.)

Output characteristics for $x_3(t) = A_1 = 4$°R., $x_4(t)$ = arbitrary upset

$y_{1m1} = -0.000091$ lb.-moles/cu. ft.	$t_{1m1} = 5$ sec.
$y_{2m1} = -0.036$°R.	$t_{2m1} = 5$ sec.
$1/\omega_1 = 30$ sec.	$\left\lvert\dfrac{y_{1m1}}{y_{1m2}}\right\rvert = 6.0$
$1/\omega_2 = 30$ sec.	$\left\lvert\dfrac{y_{2m1}}{y_{2m2}}\right\rvert = 6.0$

applying the compensating controllers C_{14}, C_{24}, C_{12}, C_{21}, K_{41}, K_{42}, K_{43}, and K_{44} and stabilizing controllers G_{fb1} and G_{fb2} to the linear model given by Equations (3) and (4). The controller constants K_{41}, K_{42}, K_{43}, K_{44}, K_{c1}, K_{c2}, T_{d1}, T_{d2}, T_1, and T_2 of Table 3 were used. The results are given in Table 4. The results summarized in Table 4 should, ideally, correspond to the results given in Table 3. These two tables are compared in Table 5 in terms of percent deviation between the two sets of results.

Except for t_{2m1}, percent errors between results of analog simulation and results obtained for perfect noninteraction for important output characteristics are between 0 and 8%. Taking into account normal error in operation of the analog computer, recorder errors, and approximations involved in reading output curves, we can take these results as definite verification that the compensating and stabilizing controllers designed on the basis of the system in the V' canonical form are operating as specified for the system in the physical P form. It should be noted at this point that addition of G_{ff1} and G_{ff2} to the system would eliminate the effect of $x_3(t)$ on $y_1(t)$ and $y_2(t)$, hence achieving complete noninteraction and perfect output control.

TABLE 4. OUTPUT CHARACTERISTICS OF THE LINEAR P FORM OF THE CONTINUOUS FLOW STIRRED-TANK REACTOR FOR THE CONTROLLER CONSTANTS AND INPUT $x_3(t)$ OF TABLE 3 AS DETERMINED BY ANALOG COMPUTER SIMULATION

$x_4(t)$, lb.-moles/ cu. ft.	$y_{1m1} \times 10^4$, lb. moles $\times 10^4$/cu. ft.	$y_{2m1} \times 10^3$, °R. $\times 10^3$	$1/\omega_1$, sec.
0.04	−0.93	−3.4	30
0.02	−0.91	−3.5	31
0.0	−0.91	−3.5	31
−0.02	−0.98	−3.7	30
−0.04	−0.94	−3.5	30

$x_4(t)$, lb.-moles/ cu. ft.	$1/\omega_1$, sec.	t_{1m1}, sec.	t_{2m1}, sec.	$\left\lvert\frac{y_{1m1}}{y_{1m2}}\right\rvert$	$\left\lvert\frac{y_{2m1}}{y_{2m2}}\right\rvert$
0.04	29	5	6	6.1	5.7
0.02	30	5	6	6.0	5.8
0.0	30	5	6	6.1	5.9
−0.02	30	5	6	6.3	5.6
−0.04	29	5	6	5.9	5.6

TABLE 5. COMPARISON OF OUTPUT CHARACTERISTICS OF TABLES 3 AND 4

% deviation of results of Table 4 from results of Table 3

$x_4(t)$ lb.-moles/ cu. ft.	y_{1m1}	y_{2m1}	$1/\omega_1$	$1/\omega_2$	t_{1m1}	t_{2m1}	$\left\lvert\frac{y_{1m1}}{y_{1m2}}\right\rvert$	$\left\lvert\frac{y_{2m1}}{y_{2m2}}\right\rvert$
0.04	2.0	6.0	0.0	3.0	0.0	20.0	2.0	5.0
0.02	0.0	3.0	3.0	0.0	0.0	20.0	0.0	3.0
0.0	0.0	3.0	3.0	0.0	0.0	20.0	2.0	2.0
−0.02	8.0	3.0	0.0	0.0	0.0	20.0	5.0	7.0
−0.04	3.0	3.0	0.0	3.0	0.0	20.0	2.0	7.0

CONCLUSIONS

The compensating controls designed above were found to decouple the linear form of the given physical system as predicted in the general theoretical development. It should be noted that work has also been carried out to test the applicability of the proposed method to nonlinear systems (*2*). However, the results of these experiments have not been presented here, since it was desired only to verify the theoretical analysis for linear systems, as previously discussed (*1*).

NOTATION

A = cooling coil heat transfer area, cu.ft. = 500 cu.ft.
A' = reaction constant, sec.$^{-1}$ = 3×10^{11} sec.$^{-1}$
C_c = coolant heat capacity, B.t.u./(lb.)(°R.) = 1 B.t.u./(lb.)(°R.)
C_p = heat capacity of reactor fluid, B.t.u./(lb.)(°R.) = 1 B.t.u./(lb.)(°R.)
E = activation energy, B.t.u./lb.-mole = 4.5×10^4 B.t.u./lb.-mole
ΔH = heat of reaction, B.t.u./lb.-mole = -2×10^4 B.t.u./lb.-mole
$Q(t)$ = feed rate, cu.ft./sec.
Q_o = steady state value of $Q(t)$ = 0.5 cu.ft./sec.
$Q_c(t)$ = coolant flow rate, cu.ft./sec.
Q_{co} = steady state value of $Q_c(t)$ = 0.2 cu.ft./sec.
R = gas constant, B.t.u./lb.-mole/°R. = 1.98 B.t.u./lb.-mole/°R.
ρ = reactor fluid density, lb./cu.ft. = 60 lb./cu.ft.
ρ_c = coolant density, lb./cu.ft. = 60 lb./cu.ft.
$T(t)$ = output temperature, °R.
T_o = steady state value of $T(t)$ = 718°R.
T_c = coolant temperature, °R. = 520°R.
$T_i(t)$ = input feed temperature, °R.
T_{io} = steady state value of $T_i(t)$ = 690°R.
U = cooling coil heat transfer coefficient, B.t.u./(hr.)(sq.ft.)(°R.) = 103 B.t.u./(hr.)(sq.ft.)(°R.)
V = reactor volume, cu.ft. = 100 cu.ft.
$X(t)$ = output reactant concentration, lb.-moles/cu.ft.
X_o = steady state value of $X(t)$ = 0.241 lb.-moles/cu.ft.
$X_i(t)$ = input reactant concentration, lb.-moles/cu.ft.
$X_{io}(t)$ = steady state value of $X_i(t)$ = 0.5 lb.-moles/cu.ft.

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